

# Indexing of fuzzy regions

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**Abstract.** This paper first exposes an algorithm that leads to fuzzy segmentation of color images. This algorithm performs, as in the watershed method, a progressive flood of the gradient image from pixels of lowest gradients. Membership degrees of pixels to regions depend on topographic distance, which takes into account both the distance to the core and the gradient norms. Geometric and colorimetric features are defined to build a region signature. A distance between fuzzy regions is then proposed, allowing ranking fuzzy regions by similarity. Applications concern region indexing and retrieval.

## 1 Introduction

Image indexing rarely use regions, probably because automatic or semiautomatic segmentation is very difficult to obtain. Problem is even more difficult when indexing generalist databases where images and regions have various sizes. In pre-attentive vision, our visual system perceives some zones with their average color, their coarse shape, their size, with respect to the rest of the image. To recognize objects, it is not necessary for regions to be exactly segmented or for contours to be exactly positioned. We propose to perform a coarse but automatic segmentation into regions which are fuzzy sets. A region signature is then computed and a distance between signatures allows to retrieve regions which are similar to a request region.

In this paper we only display results of region retrieval within an image. Of course, the distance we propose can be applied for pattern recognition or for image retrieval from partial request.

Few image retrieval systems use regions. QBIC [3] system asks the user to make part of the segmentation manually. Wood and al. [10] system begins with a coarse and incomplete segmentation of the image and then calculates a few features (3 colorimetric features and the coordinates of the center of gravity) of the extracted regions. For a region pointed by the user, a vector of features is used as input of the system of object search in a base of photographs. Stricker and Dimai [8] build 5 fuzzy regions by image, one in the center and the others covering the 4 corners of the image, arguing that most images present a central object on a background. Index is then constituted by 9 features

by region, means and covariances of color component in Lab space, each feature being computed by weighting every pixel by its membership degree.

## 2 Fuzzy segmentation

Although the expression “fuzzy segmentation” is sometimes used, it is not clearly defined. We propose the following definition :

Let  $\Omega$  be a finite referential (set of  $N$  sites). A fuzzy region is a fuzzy set of  $\Omega$  defined by a mapping from  $\Omega$  to  $[0, 1]$ . A fuzzy segmentation of  $\Omega$  is a set of  $M$  fuzzy regions  $R_j$  whose supports are included in  $\Omega$  and defined by the two following axioms :

$\mu_{R_j}(s)$  is the membership degree of site  $s$  to region  $R_j$

$$(a) \quad \forall s \in \Omega, \forall R_j, j = 1, \dots, M, \mu_{R_j}(s) \in [0, 1]$$

$$(b) \quad \forall R_j, j = 1, \dots, M, \sum_{s \in \Omega} \mu_{R_j}(s) \in ]0, N[.$$

In this section, we rapidly explain our algorithm of fuzzy segmentation, a more complete version, with more results can be found in [5]. It starts from an image of gradient norms obtained by Di Zenzo’s algorithm [2]. It performs a region growing by simulating the flood of the image as the watershed algorithm [4] [9].

Every local minimum of the gradient norm is a seed of a basin. This leads to a very big number of basins. These basins are then merged, according to criteria of size and relative depth of the basins[1]. Merged basins constitute the fuzzy regions. Finally membership degrees to the fuzzy regions are computed.

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## 2.1 Creation of the initial basins

This first stage applies watershed algorithm [9], combined with a criterion of basin merging proposed in [1].

Pixels are processed in increasing order of their values (gradient norms). Sites of level  $h$  are either integrated into an already existing basin, if they constitute a connected extension of it, or labeled as a new basin; they will in this case constitute the core of the new basin.

When two basins get in contact, their areas and depths are checked, and if one of these features is lower than fixed thresholds, the corresponding basin is affected with a pointer towards the other one, which will absorb it.

At the end of this first stage, we have an over-segmented image in many basins.

## 2.2 Fuzzy regions

In the second stage, membership degrees to the initial basins are computed, then merging is checked and finally membership degrees of absorbed basins are adjusted.

Membership degrees of sites are computed from topographic distance to region's core [5].

Let  $f$  be the image of gradient norms defined on  $\Omega$  and  $\mu_R(x)$  be the membership degree of site  $x$  to region  $R$ .

The topographic distance between  $p$  and  $q$  of  $\Omega$  is defined by Eq (1) where the minimum is taken on all 4-connected paths  $\pi$  linking  $p$  and  $q$ .

$$\pi = \{p = p_1, p_2, \dots, p_{n_\pi} = q\}, p_i \in \Omega$$

$$T(p, q) = \min_{\pi} \sum_{i=2}^{n_\pi} k |f(p_i) - f(p_{i-1})| + d_1(p_i, p_{i-1}) \quad (1)$$

where  $d_1$  is a distance in  $\Omega$ , which in the simplest case of 4-connectivity equals 1 and  $k$  is a scale parameter, which allows to balance gradient norms and spatial distance between sites.

Membership degrees of basins' cores are set to 1, fuzzy regions are extended from cores until membership degrees equal zero. Membership degrees are computed as follows :

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For each basin B
  for each site s of B
    for each neighbor v (in 4-connectivity) of s
       $\mu = \mu_B(s) - (k \cdot |f(v) - f(s)| + 1)$ 
      if  $\mu > \mu_B(v)$  then  $\mu_B(v) \leftarrow \mu$ 
    end for
  end for
End for

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When two basins merge, a penalty is applied to all pixels of the absorbed basin, in order to maintain the highest membership degrees to core pixels of the

absorbing basin. The membership degree of every pixel  $s$  of an absorbed basin is so modified :

$$\mu(s) \leftarrow \mu(s) - |h_B - h_A|$$

where  $h_B$  (resp.  $h_A$ ) is the bottom's level of the basin containing  $s$  (resp. of the absorbing basin).

If both cores have the same level, the penalty equals zero and the region's core is non-connected.

Figure 1 displays a color image with three fuzzy regions. The algorithm builds 64 fuzzy regions, each of them corresponding more or less to one entity of the image (gray levels correspond to membership degrees).

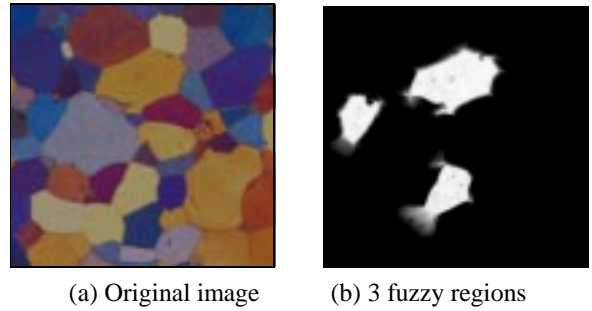


Fig. 1 : Color image of aluminum

The advantage of this algorithm is that it provides closed regions, constrained by contours. Region edges are accurate when they separate areas of different colors. Impulse noise is outlined. When transitions are slow, regions spread out and overlap each other.

A generalist database of over 1000 images has been automatically segmented, with  $k = 2$ , depth threshold = 3. The area threshold is tuned so that the number of regions is within the interval [5, 20]. Two examples are displayed Fig. 2 (merged basins are displayed).

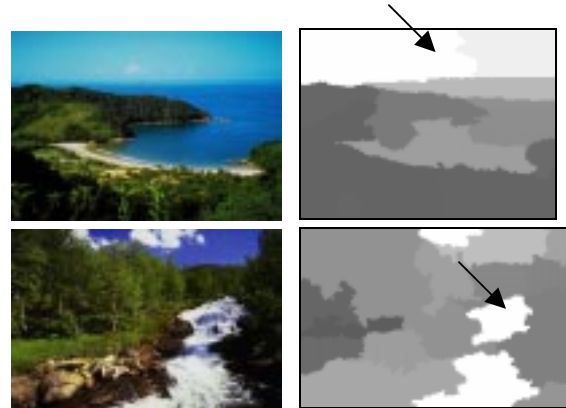


Fig. 2 : Two images of the database, automatically segmented, regions ranked by similarity

## 3 Features of fuzzy regions and region signature

Since belonging of pixels to regions is measured by the membership degrees, this information must be taken

into account in region signature. Rosenfeld [6] extended the definitions of classic geometrical features to fuzzy sets. The principle is to weight the contribution of each pixel by its membership degree.

So the area of fuzzy region R is  $\sum_{s \in R} \mu(s)$ , the height is

$\sum_y \max_x \mu(x, y)$  and the width is  $\sum_x \max_y \mu(x, y)$ . The

perimeter is  $\sum_{\substack{i,j,k \\ i < j}} |\mu_i - \mu_j| L_{ijk}$  where  $L_{ijk}$  is the length of

$k^{\text{th}}$  edge separating levels  $\mu_i$  et  $\mu_j$  ( $\mu$  is piecewise constant).

We also used the compactness = area/perimeter<sup>2</sup> and the rectangularity = area/(height×width). The first one is not bounded by  $1/4\pi$  like with crisp sets. But it is invariant to changes of scale and to rotations. The second one is lower or equal to 1 and is invariant to changes of scale, but not to rotations.

With the same principle, we have defined colorimetric features and a colorimetric distribution of fuzzy regions.

The colorimetric mean of feature c is :

$$\text{mean}_c = \frac{\sum_{s \in R} \mu(s)c(s)}{\sum_{s \in R} \mu(s)}$$

The distribution of a colorimetric feature is computed by adding the membership degrees of the pixels of the various classes of the distribution. So pixels with weak membership degrees – belonging to transitions or outliers inside a region – have little influence on the distribution shape.

To take into account both geometrical features and colorimetric features, we build a signature composed on one hand of 3 geometrical features, area, rectangularity and compactness (see [6] for details) and on the other hand the color distribution in HSV space split into 162 classes based on 18 hues, 3 intensities, and 3 saturations. The region signature consists in 165 features.

A simple application is to extract from an image the most similar regions to a request region designed by the user. A distance between every region - or target region - and the request region is computed.

#### 4 Similarity measure between fuzzy regions.

The use of features of different kinds led us to formulate a measure of similarity using a merging operator.

Let C be a target region and  $C^g = \{C_i^g, i = 1, 2, 3\}$  be the set of its geometrical features (area, rectangularity, compactness).  $C^c = \{C_j^c, j = 1, \dots, 162\}$  is the color

distribution with  $\sum_j C_j^c = 1$ . The set of these two vectors

forms the signature of the target region.

The signature of request region R is  $\{R^c, R^g\}$ .

The system separately computes a geometrical distance and a colorimetric distance. These two distances are then merged. Of course they must have similar dynamics to be merged.

The distance between color features is simple :

$$D_{col}(R, C) = \frac{1}{2} \sum_{j=1}^{162} |R_j^c - C_j^c|$$

Normalization is insured by the division by 2. This distance is maximal when both distributions contain the single value 1, positioned on two different classes.

The distance between geometrical features defined by the simple  $L_1$  distance infers a normalization problem : a normalization with regard to a maximal measurement on the image, or a fortiori on a set of images, can create distortions. For example, a region of area 100 must be at equal distance to a region of area 50 and to a region of area 200, and this is not true with  $L_1$  distance.

So we propose to use the ratio of geometrical features of C and R. To get a normalized value, we used the following function.

$$f(x) = \begin{cases} 1-x, & \text{if } x \in [0,1] \\ 1-\frac{1}{x}, & \text{if } x \in ]1,+\infty[ \end{cases}$$

The distance between geometrical features  $F_1$  and  $F_2$  is  $d_g(F_1, F_2) = f(F_1 / F_2)$ . It is easy to show that  $d_g$  is a distance .

The geometric distance between two fuzzy regions R

and C is  $D_{geo}(R, G) = \sum_{i=1}^3 \alpha_i f\left(\frac{R_i^g}{C_i^g}\right)$  with  $\sum \alpha_i = 1$

Weights  $\{\alpha_i\}$  may be tuned by the user to refine the research criteria, by removing or increasing one of them.

To merge distances between geometrical features and colorimetric features, we have chosen the operator *max* because its behavior is severe. Finally the distance between fuzzy regions R and C is :

$$D(R, C) = \max(D_{geo}(R, C), D_{col}(R, C))$$

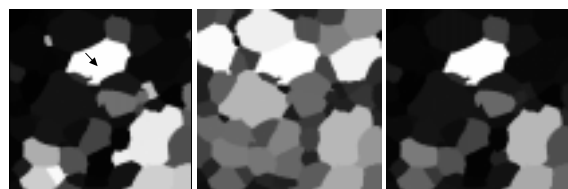
#### 5 Results of region research based on similarity

In the displayed results, the search for similar regions is only performed within the image. The user chooses a region of the image and the features on which he wishes to make his search. Since it is impossible to show all fuzzy regions (they overlap), system displays the basins

after merging. Results display every basin, with a gray level proportional to the distance with the request region ( the brighter, the closer).

For example in Fig 2, the user chose a part of the sky for the first example and a part of the river for the second example and the colorimetric feature. The system displays regions ranked by decreasing similarity. The system found 4 regions of sky (for the first one) and 2 regions of river (for the second one) as the most similar to request regions.

In Fig. 3, the request is a large dark yellow region. When using only the colorimetric features, the yellow regions are the closest to the request, dark yellow regions are closer than light yellow ones. When using only the area, regions are ranked by size. Merging both distances gives the large yellow regions. Figure 4 addresses the problem of object occlusion. In this special case of circular objects, rectangularity added to color allows to retrieve partially occluded objects, even if half the object is missing.

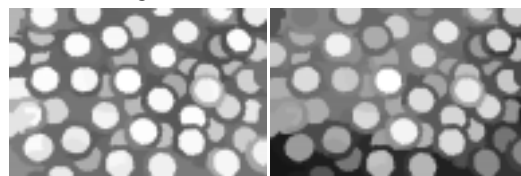


(a) Color (b) Area (c) Merging

Fig. 3 : Regions ranked by decreasing similarity to a request region, marked with an arrow (from white to black)



(a) Image of coins (b) Color



(c) Rectangularity (d) Merging

Fig. 4 : Similarity using color and rectangularity for retrieval of partially occluded objects

## 6 Conclusion

We have proposed an algorithm of fuzzy region extraction which can be used in pattern recognition or in image retrieval. The algorithm is automatic, since regions are dynamically created during the process. It needs to tune three parameters, which are not critical. Two of them can be fixed to segment a whole database. The area threshold is tuned by dichotomy according to a coarse expected number of regions.

Similarity measures based on color or geometric features of fuzzy regions are promising. Of course better distances can be used and a relevance feedback will automatically tune the weight of each feature. Another interest of a signature based on regions is the ability to include spatial relationships between regions, in order to manage requests composed of several regions.

## 7 References

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