DETECTION OF HIGH FREQUENCY REGIONS IN MULTIRESOLUTION

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ABSTRACT

We propose a method for the detection of high frequency regions using multiresolution analysis and orientation tensors. A scalar field representing multiresolution edges is obtained. Local maxima of this scalar space indicate regions having coincident detail vectors in multiple scales of a wavelet decomposition. This is useful for finding edges, textures, collinear structures and salient regions for computer vision methods. The image is decomposed into several scales using the Discrete Wavelet Transform (DWT). The resulting detail spaces form vectors indicating intensity variations which are adequately combined using orientation tensors. The multivariate data of the resulting tensor field provides fair estimations of high frequency regions. Using these tensors, a positive scalar is computed for each original image pixel. Our results show that this descriptor indicates areas having relevant intensity variation in multiple scales.

Index Terms— high frequency detection, multiresolution analysis, multiresolution edges, orientation tensor.

1. INTRODUCTION

The evaluation of high frequencies in an image is an important task for several applications in computer vision, computer graphics and image processing. Objects in a scene are mainly distinguished by the contrast of their borders against a background. From a signal processing point of view, this can be seen as brightness variation with multiple frequencies.

However, object and background areas can be arbitrarily complex. One way of estimating salient regions is to use multiresolution to capture global and local brightness variations. Even in a non-redundant wavelet decomposition, local and global borders occurring in the same region may carry useful information. The problem lies in combining this global information into a single image. In this way, orientation tensors can capture the multivariate information of several scales and color channels [1].

For image segmentation, Belkasim et al. [2] uses a multiresolution image analysis scheme based on extracting all objects in the image using their borders or contours. The size of the contour can then be used to define the level of resolution and hence the extent of the analysis.

Shih et al. [3] argue that edge extraction based only on a gradient image will produce bad results with noise and broken edges. In order to solve this problem, they combine an edge detector with a multiscale edge tracker.

By combining both orientation tensor and multiresolution analysis, one may have a scalar descriptor of high frequency regions [4]. High values of this scalar space indicate regions having coincident detail vectors in multiple scales of a wavelet decomposition.

In this paper, the multivariate information contained in the tensor field is used to find coincident edges in multiresolution space. More specifically, we combine the scalar field proposed in [4] with the eigenvectors obtained, in order to find coherent edges.

2. FUNDAMENTALS

2.1. Wavelets

The wavelet transform decomposes signals over dilated and translated wavelets [5]. A wavelet is a function $\psi \in L^2(\mathbb{R})$ with a zero average:

$$\int_{-\infty}^{+\infty} \psi(t)dt = 0 \qquad (1)$$

It is normalized $||\psi|| = 1$, and centered in the neighborhood of $t = 0$. A family of time-frequency atoms is obtained by scaling ψ by s and translating it by u:

$$\psi_{u,s}(t) = \frac{1}{\sqrt{s}}\psi\left(\frac{t-u}{s}\right) \qquad (2)$$

We are interested in wavelets which form a base of $L^2(\mathbb{R}^2)$ to represent images. If we have an orthonormal wavelet basis in $L^2(\mathbb{R})$ given by ψ with the scaling function ϕ, we can use

$$\psi^1(x) = \phi(x_1)\psi(x_2),\ \psi^2(x) = \psi(x_1)\phi(x_2),$$

$$\psi^3(x) = \psi(x_1)\psi(x_2) \qquad (3)$$

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to form an orthonormal basis in $L^2(\mathbb{R}^2)$ [5].

$$\{\psi^1_{j,p}, \psi^2_{j,p}, \psi^3_{j,p}\}_{[j,p]\in Z^3} \tag{4}$$

In this paper, we define a vector $v_{j,p} \in \mathbb{R}^3$ given by the inner product

$$v_{j,p} = [I \cdot \psi^1_{j,p}, I \cdot \psi^2_{j,p}, I \cdot \psi^3_{j,p}]^T \tag{5}$$

at scale j and position $p \in I$, where I is the input image.

2.2. Orientation Tensor

A local orientation tensor is a special case of non-negative symmetric rank 2 tensor, built based on information gathered from an image. As shown by Knutsson [1], such a tensor can be produced by combining outputs from polar separable quadrature filters. Because of its construction, such a tensor has special properties and contains valuable information about said image.

From the definition given by Westin [6], orientation tensors are symmetric, and thus an orientation tensor T can be decomposed using the Spectral Theorem as shown in Eq. 6, where λ_i are the eigenvalues of T.

$$T = \sum_{i=1}^{n} \lambda_i T_i \tag{6}$$

If T_i projects onto a m-dimensional eigenspace, we may decompose it as

$$T_i = \sum_{s=1}^{m} e_s e_s^T \tag{7}$$

where $\{e_1,...,e_m\}$ is a base of $\mathbb{R}^m$. An interesting decomposition of the orientation tensor T proposed by Westin [6] is given by

$$T = \lambda_n T_n + \sum_{i=1}^{n-1} (\lambda_i - \lambda_{i+1}) T_i \tag{8}$$

where λ_i are the eigenvalues corresponding to each eigenvector e_i. This is an interesting decomposition because of its geometric interpretation. In fact, in $\mathbb{R}^3$, an orientation tensor T decomposed using Eq. 8 can be represented by a spear (its main orientation), a plate and a ball

$$T = (\lambda_1 - \lambda_2)T_1 + (\lambda_2 - \lambda_3)T_2 + \lambda_3 T_3. \tag{9}$$

A $\mathbb{R}^3$ tensor decomposed by Eq. 9, with eigenvalues $\lambda_1 \geq \lambda_2 \geq \lambda_3$, can be interpreted as following:

- $\lambda_1 >> \lambda_2 \approx \lambda_3$ corresponds to an approximately linear tensor, with the spear component being dominant.
- $\lambda_1 \approx \lambda_2 >> \lambda_3$ corresponds to an approximately planar tensor, with the plate component being dominant.
- $\lambda_1 \approx \lambda_2 \approx \lambda_3$ corresponds to an approximately isotropic tensor, with the ball component being dominant, and no main orientation present.

Consider two orientation tensors A and B and its summation $T = A + B$. After the decomposition of T using Eq. 9, the component $(\lambda_1 - \lambda_2)T_1$ is an estimate of the collinearity of the main eigenvectors of A and B.

2.3. Multiresolution High Frequency Assessment

The method proposed in [4] uses high frequency information extracted from wavelet analysis. For each scale j, a vector based on Eq. 5 is created. This vector contains the high frequency value at vertical, horizontal and diagonal directions of the image I at the position p and scale j. Symmetric rank 2 tensors are then created as

$$M_{j,p} = v_{j,p} v_{j,p}^T. \tag{10}$$

The final tensor $M_{0,p}$ is computed for each pixel of the original image using

$$M_{0,p} = \sum_{j=1}^{n_j} k_j M_{j,p} \tag{11}$$

to combine the tensors obtained at each scale j, where n_j is the number of scales and $k_j \in \mathbb{R}$ is the weight assigned to each scale, given by

$$k_j = \frac{\sum_{n=1}^{n_p} \text{Trace}(M_{j,n})}{\sum_{k=1}^{n_j} \sum_{n=1}^{n_p} \text{Trace}(M_{k,n})}, \tag{12}$$

where n_p is the number of pixels and Trace$(M_{j,p})$ is the sum of the eigenvalues of $M_{j,p}$. The trace represents the amplification driven by the tensor to the unit sphere and is a good estimator of its importance. Thus, the tensor sum is weighted by the proportion of energy of each scale in the multiresolution pyramid.

For each pixel p of the input image, its correspondent position at the current scale j is computed with subpixel precision for each resolution. The four nearest pixels in this resolution are used to compute the final tensor. The vectors $v_{j,p}$ described in Eq. 5 are computed for each of these pixels and then used to compute four spear type tensors. The final tensor $M_{j,p}$ (Eq. 10) for the subpixel position is obtained by combining these four tensors with bilinear interpolation. The pixel tensor $M_{0,p}$ is computed by combining the n_j tensors as showed in Eq. 11.

The tensors are then decomposed using Eq. 9 and their eigenvalues are extracted. The values λ_1 - λ_2 are computed and normalized. They indicate the collinearity of the interpolated tensors and provides interesting results. Color images are split into three monochromatic channels (Red, Green and Blue) and the proposed algorithm is applied to each channel separately. The tensors for each color channel are summed before eigen decomposition.

3. PROPOSED METHOD

The proposed method consists of combining the eigenvectors e_1, e_2 and e_3 of each tensor with the resulting scalar field λ_1 - λ_2. We argue that when the main direction is coincident with the variation of λ_1 - λ_2, we have a salient multiresolution region. A method similar to the proposed in [7] can be derived to extract these regions.

Since the tensor is a symmetric positive matrix, its eigensystem forms an orthonormal basis of $\mathbb{R}^3$ where e_1 represents the estimated main direction and e_1 with e_2 form the best estimated plane. Using the method presented in Section 2.3, e_1 represents the resulting gradient vector obtained from several scales. The value λ_1 - λ_2 is an estimation of their collinearity. The plane $e_1 e_2$ is the plane where the combined gradient is likely to be.

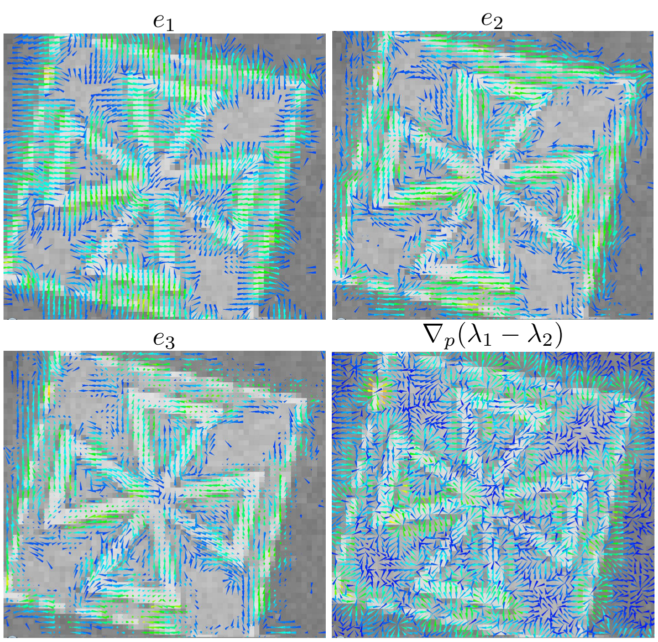

Fig. 1. Eigenvector fields and the gradient $\nabla_p(\lambda_1 - \lambda_2)$ overlayed with the input image. Fields obtained using daub2 filter and two scales.

To describe high frequency regions in multiresolution, we propose the scalar field

$$s_p = |\cos\theta| + |\cos\alpha| \tag{13}$$

where θ is the angle between e_1 and $g_p = \nabla_p(\lambda_1 - \lambda_2)$, and α is the angle between g_p and the plane $e_1 e_2$. Pixels having high values of $0 \leq s_p \leq 2$ are likely to be salient multiresolution edges. A segmentation of this scalar field using a threshold is sufficient to highlight multiresolution high frequency regions.

As an example, eigenvectors and the gradient of λ_1 - λ_2 are shown in Fig. 1, using a part of the classic Pentagon image. The vectors are overlayed with the original image using a thermal palette to indicate $\lambda_1 - \lambda_2$. Vectors with small values of $\lambda_1 - \lambda_2$ were omitted.

The complexity of the whole process is $O(n_j \cdot n_p)$, where n_j is the number of analyzed scales and n_p the amount of input pixels. Thus, this is an efficient method that can be further parallelized.

4. EXPERIMENTAL RESULTS

The Fig. 2b shows the result of $\lambda_1 - \lambda_2$ obtained for the image Fig. 2a. The response of high frequencies assessment is higher with more scales. In general, it can be noted that high frequencies occurring in the same region at different scales are highlighted by this method. The thermal coloring is a smooth transition from blue to red, where blue means absence of coincident high frequencies, and red means presence of coincident high frequencies.

The segmentation of pixels having high s_p values is also showed in Fig. 2. The threshold value 0.5 is used in all examples. In Fig. 2c, the result using daub2 filters with two scales starts to show that the upper side of the Pentagon has interesting brightness variations. Using a regular edge detector, these borders would be highlighted independently. With our method, one may see that this region has high detail energy in all used scales. With 3 scales (Fig. 2d), it becomes more evident. This may be an interesting multiresolution feature for detection systems. The response is similar using the daub3 filter with 4 scales (Fig. 2e). Noise is a problem when propagated by all scales (Fig. 2f).

The second experiment shows the time spent to apply the algorithm in color images. Fig. 3 shows the time in seconds in function of the number of scales and image size. One may see the linear behavior of the algorithm, where the slope is proportional to the number of scales. However, it is important to note that the algorithm response time may be a bottleneck in real time applications if the number of pixels is high. All experiments were performed on an Intel Core2 Duo 1.8Ghz CPU using a 32bit compiler.

5. CONCLUSIONS AND FUTURE WORKS

A method for high frequency multiresolution assessment and edge extraction was proposed. It is based on the DWT decomposition followed by detail information merging using orientation tensors. This multiresolution analysis showed to be suitable for detecting relevant edges and salient areas in an image. Due to the multivariate nature of tensors, the process can be easily applied in color images.

The experimental results show that the high frequency information can be inferred by varying the DWT filters and number of scales. Coincident frequencies in space domain are successfully highlighted. Tensor information was exploited to obtain local maxima at multiresolution edges. By tuning the number of scales, one may highlight specific high frequency

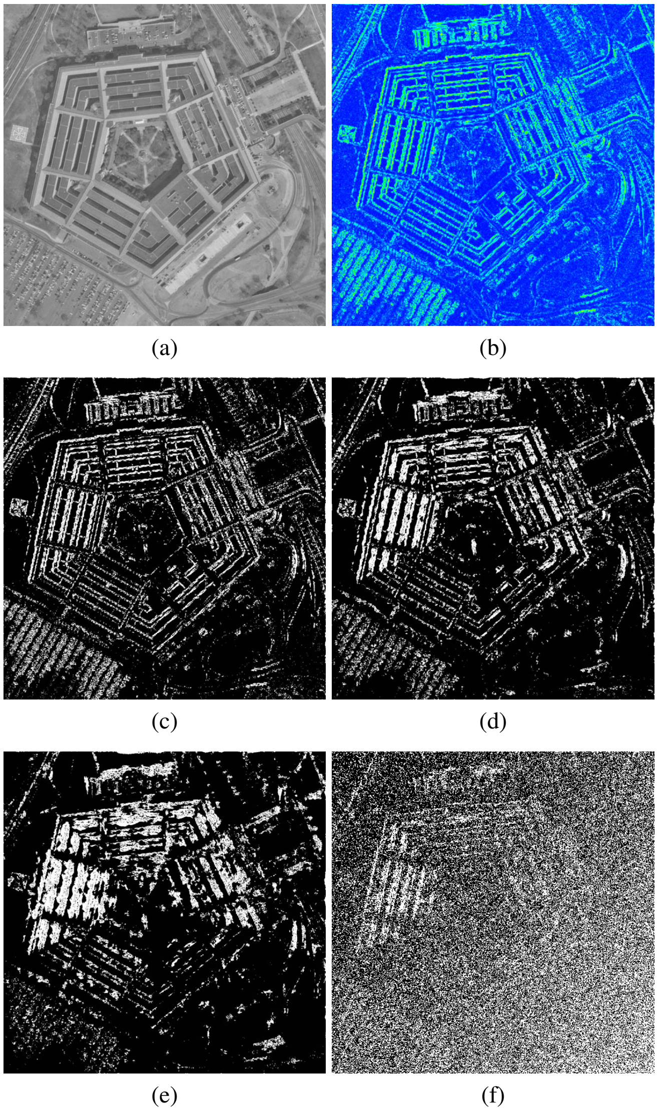

Fig. 2. (a) input image with 1024×1024 pixels. (b) λ_1 - λ_2 with daub2 and 2 scales. Results with white pixels having $s_t > 0.5$ and using: (c) daub2 and 2 scales, (d) daub2 and 3 scales, (e) daub3 and 4 scales, (f) daub3 and 4 scales with 10% of gaussian noise.

regions. As shown, the linear complexity is suitable for high performance processes. The presence of random noise in the image may generate bad results. Problems arise when noisy borders are detected on several scales. As a result, errors in these regions are propagated in the tensor accumulation process. A previous low-pass filtering may reduce this problem.

The $\lambda_1 - \lambda_2$ scalar field is one of the most used orientation alignment descriptors. However, other relations can be extracted from final pixel tensors. The effects of varying wavelet filters and number of scales are still unclear and need further investigation. Scalar fields with zero crossings can also be derived. This is also a promising line for future works.

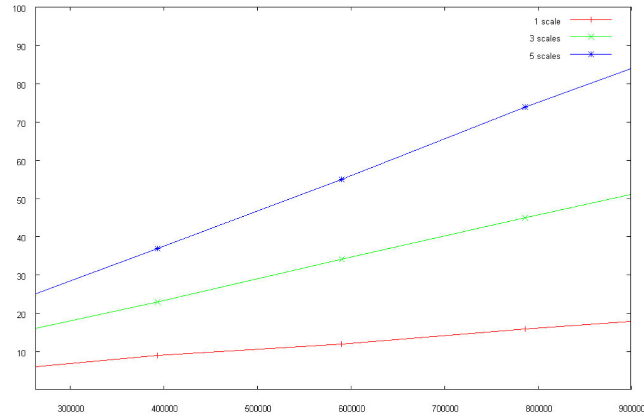

Fig. 3. Evaluation of the running time in function of the number of scales and amount of pixels of a color image.

The discrete wavelet transform and the tensor summation can be easily parallelized. The use of rising technologies like gpGPUs and multicore CPUs turns this method attractive for high performance applications.

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